Optics Communications 89 (1992) 403–409
North-Holland

OPTICS
COMMUNICATIONS

Laser-induced grating spectroscopy of electron–phonon interaction in metallic and high-temperature superconductors

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Received 2 September 1991

We propose a new spectroscopical method for the investigation of electron–phonon (EP) interaction on subpicosecond time scale in metallic and high-temperature (HT) superconductors by using a tunable laser beam, without requiring femtosecond laser pulses or optical detectors. The method is based on the self-diffraction of a laser beam from a laser-induced temperature moving gratings, and on the close connection between EP interaction and thermal relaxation of hot carriers. The theory of the method has been developed in detail. Two regimes have been considered: (a) the regime of weak heating of the electron system and (b) the regime of strong heating of the electron system. It has been shown that for regime (a) the proposed method permits to measure the EP interaction constant that determines the superconducting transition temperature in the metallic superconductors. In regime (b) the suggested method permits both the determination of the EP interaction constant and the direct measurement of the dependence of thermal electron relaxation on temperature. Such measurements can provide pertinent information about the contribution of the EP interaction to the HT superconductivity.

1. Introduction

Electron–phonon (EP) interaction plays a main role in metallic [1] and possibly an important role in the high-temperature (HT) superconductors [2]. Therefore, measurements of EP interaction in superconductors are very important. In particular, such measurements can provide information about the contribution of EP interaction to the HT superconductivity [3,4].

The topic of EP interaction is closely related to the thermal relaxation of hot electrons (or other carriers). Allen [5] was able to relate the relaxation rate of electron temperature T_e to the factor $\lambda\langle\omega^2\rangle$, which is of great interest in the theory of the superconducting transition temperature T_c:

$$\frac{\partial T_e}{\partial t} = -\frac{3\hbar}{\pi k_B T_e}\lambda\langle\omega^2\rangle(T_e - T_i) \equiv \tau_e^{-1}(T_e - T_i)\,. \tag{1}$$

Here $\langle\omega^2\rangle$ is the second moment of the phonon spectrum, T_i is the lattice temperature, τ_e is the relaxation time of the electron temperature connected with the energy change between electrons and lattice.

In recent time resolved studies on thermal modulation of the reflectivity and the transmissivity using femtosecond laser pulses, nonequilibrium electron and lattice temperatures have been observed [3,4,6–9], and EP coupling constant λ has been determined both in metallic superconductors [7] and in cuprates (nonsuperconducting La_2CuO_{4+y} [3] and superconducting $YBa_2Cu_3O_7$ [4]). These pump-probe methods require femtosecond laser pulses (complex systems and expensive), and the knowledge of both the lattice (C_i) and electronic ($C_e = \bar{\gamma}T_e$) specific heats and also initial $T_e(0)$ and final $T_e(\infty)$ electron temperatures [7]. The latter can decrease the accuracy of the measurements.

Besides, ultrashort light pulses can excite nonequilibrium phonons via impulsive excitations [10–12] (in particular, "giant" vibrations [10]). In this case there is no local equilibrium in phonon system, and the consideration on the basis of phonon temperature (eq. (1)) is not correct. Thus, other independent methods of investigation are desirable.

We propose here a new method for the investi-

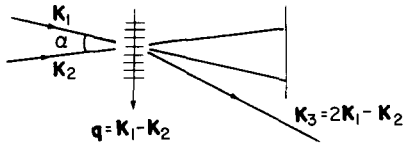

Fig. 1. The scheme describing the principles of the method. Two excitation beams $k_1\omega_1$ and $k_2\omega_2$ produce a transversely moving thermal grating. A beam $k_1\omega_1$ is self-diffracted and frequency is shifted by the moving thermal grating.

gation of EP interaction in metals and superconductors with femtosecond time resolution by operating in the frequency domain using tunable lasers. The method is based on laser-induced moving gratings and does not demand ultrashort laser pulses. In the regime of weak heating the electron system, the method permits the determination of EP interaction constant without the knowledge of C_i, C_e, $T_e(0)$ and $T_e(\infty)$. In the regime of strong heating of the electron system the proposed method, unlike femtosecond pump-probe methods [3,4,6–9], permits direct observation and measurement of the dependence of thermal electron relaxation on temperature.

Before laser-induced grating methods based on steady-state three-wave mixing were used for the investigation of electronic transitions in dye molecules and semiconductors [13–20].

The optical scheme describing the principles of methods is shown in fig. 1. Two laser beams with different frequencies $\omega_1 \neq \omega_2$ (ω_1=const., ω_2=var.) and wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ produce a nonstationary intensity distribution in the medium under investigation. This intensity exhibits a wavelike modulation with a grating vector $\boldsymbol{q}=\boldsymbol{k}_1-\boldsymbol{k}_2$ and a frequency $\Omega=\omega_1-\omega_2$. The wavelike modulated light intensity changes both the temperature and the optical properties of the material in the interference region resulting in a moving grating structure.

If this wavelike modulation is slow in comparison to the relaxation time, τ of the material temperature ($\tau \ll \Omega^{-1}$) the latter is synchronized in time with the intensity change, and the thermal grating amplitude does not decrease. If $\Omega \geqslant \tau^{-1}$, the material temperature is not synchronized in time with the intensity modulation, and the grating structure becomes less contrast, and therefore the grating amplitude decreases. The grating effectiveness measured by self-diffraction of waves ω_1 falls down for this case. In the moving grating method a comparison of the relaxation velocity of the material temperature with the motion velocity of the grating that is proportional to the frequency detuning is produced.

Any laser-induced grating method is based on a specific model of the interaction of the electromagnetic radiation with the investigated object. Below we develop the theory of the method. We shall calculate the expected dependence of three-wave mixing signal $\boldsymbol{k}_3=2\boldsymbol{k}_1-\boldsymbol{k}_2$, $\omega_3=2\omega_1-\omega_2$ on the frequency detuning Ω due to the thermal grating created in a thin film by waves, $\omega_1\boldsymbol{k}_1$ and $\omega_2\boldsymbol{k}_2$. The equations for the temperature distribution in a sample can be written in the form [21,22]

$$\frac{\partial T_e}{\partial t} = \frac{\kappa_e}{C_e}\frac{\partial^2 T_e}{\partial \boldsymbol{r}^2} - \tau_e^{-1}(T_e - T_i) + \frac{f(\boldsymbol{r},t)}{C_e}, \qquad (2)$$

$$\partial T_i/\partial t = \tau_i^{-1}(T_e - T_i), \qquad (3)$$

where $\tau_i = \tau_e C_i/C_e$ is the relaxation time of lattice temperature, $f(\boldsymbol{r},t)$ is the heat production per unit volume per unit time by the light waves $\omega_1\boldsymbol{k}_1$ and $\omega_2\boldsymbol{k}_2$,

$$f(\boldsymbol{r},t) = F_0 + \{F_1 \exp[-i(\Omega t - \boldsymbol{q}\cdot\boldsymbol{r})] + \text{c.c.}\}, \qquad (4)$$

$$F_0 = \chi(J_1 + J_2), \qquad |F_1| = \chi(J_1 J_2)^{1/2},$$

$$\chi = \hbar\omega\, 2kb(1-R)\exp(-2b\boldsymbol{k}_0\cdot\boldsymbol{r}),$$

R is the reflectivity, J_i is the power density of waves incident on the surface (in terms phot/cm^{2}s), $k_0 \equiv k_{01} = \omega_1/c \approx \omega_2/c$, $\omega \equiv \omega_1 \approx \omega_2$, $2kb$ is the absorption coefficient.

The term $(\kappa_e/C_e)\,(\partial^2/\partial \boldsymbol{r}^2)$ describes the temperature change due to the heat conduction (κ_e is the factor of electronic heat conduction). The temperature will change in two perpendicular directions: in the direction of the grating vector $\boldsymbol{q}$ and the direction of the propagation of waves $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ ($\boldsymbol{k}_1 \approx \boldsymbol{k}_2$). As to the temperature change in direction $\boldsymbol{k}_1$ ($\boldsymbol{k}_2$), we shall consider the film as thermally thin [22]. By thermally thin, we mean that during the time scale of interest t' the back surface of the film reaches approximately the same temperature as the front surface on which the radiation is incident. This situation will hold for the lattice if $L^2 C_i/(4\kappa_e) \ll t'$ [22] and for the electronic system if $L^2 C_e/(4\kappa_e) \ll t'$, where L is the thickness of the film. For $L \sim 10^{-5}$–10^{-6} cm and $C_e/\kappa_e \sim 0.01$–0.1 s cm^{-2} [21], we have

$L^2C_e/(\kappa_e) \sim 2.5\times(10^{-14}$–$10^{-13})$ s. Besides, the temperature change in direction $\boldsymbol{k}_1$ will not be large in any case, as we consider film as semitransparent (see below). Thus, we shall consider only temperature changes in $\boldsymbol{q}$ direction. The regime of weak heating electronic system has been studied in sec. 2, and the regime of strong heating electronic system has been considered in sec. 3.

2. Weak heating electronic system

We shall look for the solution of eqs. (2) and (3) in the form

$$\delta T_{e,i} = T_{e1,i1} \exp[i(\boldsymbol{q}\cdot\boldsymbol{r}-\omega t)] + \text{c.c.} \tag{5}$$

considering in this issue the light induced temperature changes of δT_e and δT_i are small in comparison with their equilibrium values T_e and T_i. For this case we obtain

$$T_{e1} = F_1(\tau^{-1}-i\Omega)/C_e\Delta\,, \qquad T_{i1} = F_1/(\tau_i C_e \Delta)\,, \tag{6}$$

where

$$\Delta = \tau_e^{-1}\tilde{T}^{-1} - i\Omega(\tau_e^{-1}+\tau_i^{-1}+(\tau_i/\tau_e)\tilde{T}^{-1}) - \Omega^2\,,$$

$$\tilde{T}^{-1} = q^2\kappa_e/C_i\,.$$

From eq. (6) we derive the ratio of the electron and the lattice subsystem amplitude contributions to thermal grating

$$T_{i1}/T_{e1} = (1-i\Omega\tau_i^{-1})^{-1}\,. \tag{7}$$

It follows from eq. (7) that for small detunings $\Omega < \tau_i^{-1}$ these contributions are equal, however, for large detunings $\Omega > \tau_i^{-1}$ the electron contribution is bigger. Thus, it is possible to drive by the amplitudes of both the electron and lattice temperature gratings by frequency detuning.

The Maxwell equation for the vector-potential of the signal wave $\boldsymbol{k}_3$ can be written as

$$\frac{\partial^2}{\partial z^2}A_3 + k_{03}^2(n_3+ib_3)^2A_3 = -\frac{4\pi}{c}\mu j^{(3)}\,, \tag{8}$$

where n_3+ib_3 is the complex refractive index, including the effect of linear absorption, z is the direction of propagating wave $\boldsymbol{k}_3$, μ is the magnetic permeability, $j^{(3)}$ is the nonlinear current density. The positive frequency component of nonlinear source in the right hand of eq. (8) can be expressed by temperature gradient (5)

$$j^{(3)+} = (\omega_1^2/4\pi c)(r_eT_{e1}+r_iT_{i1})\exp[i(\boldsymbol{q}\cdot\boldsymbol{r}-\Omega t)]A_1^+ \times \exp[-\omega_1 t + i\boldsymbol{k}_{01}\cdot(n_1+ib_1)\boldsymbol{r}]\,,$$

where $r_{e,i} \equiv \partial\epsilon/\partial T_{e,i}$, ϵ is the dielectric constant.

Solving eq. (7) with the boundary condition $A_3^+(z=0)=0$, we obtain

$$\frac{|A_3^+(z)|^2}{|A_1^+(0)|^2} = \frac{\mu^2k^2}{n^2+b^2}|r_e|^2\left|T_{e1}+\frac{r_i}{r_e}T_{i1}\right|^2 \times z^2\exp(-2kbz)\,. \tag{9}$$

The factor

$$|T_{e1}+r_iT_{i1}/r_e|^2 \equiv |I(\Omega)|^2|F_1/C_e|^2\,,$$

where

$$I(\Omega) = [\tau_i^{-1}(1+r_i/r_e)-i\Omega]/\Delta\,, \tag{10}$$

describes the dependence of the signal power on the frequency detuning Ω. For metals at room temperatures [21]: $\tau_i \sim 10^{-10}$ s, $\tau_e \sim 10^{-12}$ s, $\kappa_e/C_i \sim 0.1$–1.0 cm^2/s and $\tilde{T}^{-1} = q^2\kappa_e/C_i \sim 10^6$ s^{-1}, as for optical frequencies ($\omega/2\pi c \sim 20000$ cm^{-1}) and $\alpha \sim 1°$, $|q| \sim 10^3$ cm^{-1}. So we have

$$\tau_e^{-1} \gg \tau_i^{-1} \gg \tilde{T}^{-1}\,. \tag{11}$$

For these conditions we obtain from eq. (10)

$$I(\Omega) + \frac{\tau_i^{-1}(1+r_i/r_e)-i\Omega}{(\Omega+i\tilde{T}^{-1})(\Omega+i\tau_e^{-1})}\,. \tag{12}$$

The factor $I(\Omega)$ is proportional to the amplitude of moving temperature grating which is produced by pump waves $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, and modulates the optical properties of material. The dependence of the factor $|I(\Omega)|$ on the detuning Ω (for conditions (11)) is shown in fig. 2. For small detuning $|\Omega| \ll \tau_i^{-1}$ (ranges I and II)

$$I(\Omega) \approx i(\tau_e/\tau_i)(1+r_i/r_e)/(\Omega+i\tilde{T}^{-1})\,.$$

The grating amplitude begins to decrease for detuning $|\Omega| \geqslant \tilde{T}^{-1}$. Thus $\tilde{T}$ controls the relaxation time of equilibrium thermal grating.

For larger detunings $\tilde{T}^{-1} \ll |\Omega| \sim \tau_i^{-1}|1+r_i/r_e|$ (range III)

$$I(\Omega) \approx -(\tau_e/\tau_i)[1+i(1+r_i/r_e)/\Omega]\,.$$

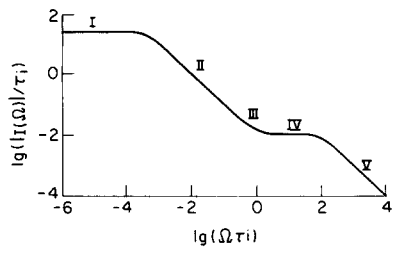

Fig. 2. The dependence of the factor $|I(\Omega)|$ on frequency detuning Ω for $\tau_i/\tau_e=100$, $\tau_i/\tilde{T}=4\times10^{-4}$.

As seen in fig. 2, the slope of the curve as a function of Ω is smaller for range III than for range II. This is due to the formation of a non-equilibrium thermal grating due to the temperature difference between the electron and the lattice subsystems.

For detunings $|\Omega| \gg \tau_i^{-1}|1+\tau_i/\tau_e|$ (range IV and V)

$$I(\Omega)=(i\Omega-\tau_e^{-1})^{-1}. \tag{13}$$

In range IV ($\tau_i^{-1}|1+\tau_i/\tau_e| \ll |\Omega| < \tau_e^{-1}$) the decrease of the equilibrium contribution to the grating compensated by an increase of the nonequilibrium contribution since the electron temperature is synchronized in time with intensity modulation. In the range V ($|\Omega| > \tau_e^{-1}$) the electron temperature already is not synchronized in time with the intensity modulation, and $|I(\Omega)| \approx 1/|\Omega|$. The tangent of the curve slope is equal to -1 for ranges II and V.

As it follows from eq. (9), the optimal thickness of a sample is $L_{opt}=(kb)^{-1}$. We shall estimate the expected signal value for the $YBa_2Cu_3O_7$ film of the optimal thickness using experimental values [23] for $\epsilon=\epsilon'+i\epsilon''$ for a photon energy $\hbar\omega\approx2$ eV and the pump-probe data on measuring change in reflection $YBa_2Cu_3O_7$ $\Delta R/R=0.15\,\Delta\epsilon'+0.25\,\Delta\epsilon''$ [4] for $\lambda=630$ nm. The change in R was $\Delta R/R\approx15\%$ for heating electron system $T_e\sim10^3$ K[#1]. For $|T_{e1}+(r_i/r_e)T_{i1}|\sim10$ K we obtain the following value from formula (9): $|A_3^+(L_{opt})|^2/|A_1^+(0)|^2\sim10^{-5}$.

As it follows from eqs. (4) and (6), the heating $|T_{e1}|\sim10$ K corresponds to the power density $(1-R)\sqrt{J_1J_2}\sim10^{26}$ phot/cm^{2}s, where we used the value τ_e ($T=300$ K) $\sim10^{-13}$ s corresponding to the experimental data $\lambda\langle\omega^2\rangle\simeq150$ meV [4] for $YBa_2Cu_3O_7$. Such pulses of duration 10^{-12} s will increase the film temperature only by ~1 K. Thus, it is possible to investigate HT superconducting samples by laser-induced grating spectroscopy without destruction of the superconductivity.

In conclusion of this issue we shall give values of inverse electron relaxation times τ_e^{-1} for $T=300$ K, calculated by data about parameters $\lambda\langle\omega^2\rangle$ for metallic [7] and HT [4] superconductors (in the parenthesis the superconducting temperatures indicated in [K]). See table 1.

3. Strong heating electron system

The value of the ratio between the signal and the probe intensities can be increased in the case of strong heating electron system, when $T_e \gg T_i$. For this case and for $t \ll \tilde{T}$, eq. (2) (or eq. (1)) can be written as

$$T_e\frac{\partial T_e}{\partial t}=-\frac{\alpha}{\bar{\gamma}}T_e+\frac{f(r,t)}{\bar{\gamma}}. \tag{14}$$

The steady-state solution of the nonlinear differential eq. (14) can be written in the form (see appendix)

[#1] As the initial temperature in experiments [4] is 55 K $< T_c$, it is possible that the heating electron system is not the only contribution to the observed value of ΔR. However, we suppose that this contribution is of the same order as the observed value of $\Delta\epsilon$.

Table 1
Inverse electron relaxation times.

	V_3Ga (16.8)	NbN (16.0)	Pb (7.2)	Ti (0.4)	Nb (9.5)	V (5.4)	W (0.0154)	Cr	Au	Cu	$YBa_2Cu_3O_7$
$\lambda\langle\omega^2\rangle$ [meV^2]	370	640	45	350	320	280	112	128	23	29	150
$\tau_e^{-1}/2\pi c$ [cm^{-1}]	110	190.3	13.4	106	95.5	83.3	33.3	38	6.9	8.5	45

$$T_e = T_0 + \{T_1 \exp[i(\Omega t - \boldsymbol{q}\cdot\boldsymbol{r})] + \text{c.c.}\},$$
$$T_0 = F_0/\alpha, \quad T_1 = T_0^{-1}[\tau_e^{-1}(T_0) - i\Omega]^{-1} F_1/\bar{\gamma}, \tag{15}$$

where $\tau_e(T_0) = \bar{\gamma} T_0/\alpha$ is the new value of the electronic temperature relaxation time that is the function of the heat production f.

So, we can use the formulae of the former section, where we must substitute for τ_e by nonequilibrium value $\tau_e(T_0)$.

In the case of strong heating electron system: $|T_1| \sim$ 500 K, the previous value of signal for $YBa_2Cu_3O_7$ strongly increases: $|A_3^+ (L_{opt})^2 / |A_1^+(0)|^2 \sim 10^{-2}$.

It is obvious that for obtaining sufficiently large light intensity J_3 and preventing film overheating it is necessary to use a pulse pump (with pulse duration $t_p > \tau_e(T_0)$). Therefore, in an experiment it is convenient to record the energy magnitude of signal $\boldsymbol{k}_3\omega_3$, $E_3 \sim \int_{-\infty}^{\infty} dt |A_3^+(t)|^2$. Using eqs. (9) and (15) and considering that pulse shapes of the pulses J_1 and J_2 are identical, we obtain

$$E_3 \sim \int_{-\infty}^{\infty} dt \frac{\vartheta^3(t)}{1 + \nu^2 \vartheta^2(t)},$$

where $J(t) = J_{max}\vartheta(t)$, $\nu = \Omega\bar{\gamma}F_{0max}/\alpha^2$.

In this connection the question comes up about the connection between the signal energy $E_3(\Omega)$ and the signal intensity $J_3 \sim [\tau_e^{-2}(T_0) + \Omega^2]^{-1}$ for the nonlinear regime. For rectangular pulses (or about rectangular) $E_3(\Omega) \sim [\tau_e^{-2}(T_0) + \Omega^2]^{-1}$. However, in the general case, it is necessary to integrate the signal using eq. (15) to obtain the dependence $E_3(\Omega)$ taking into account the real pulse shape. For example, for two sided exponential pulses $\vartheta(t) = \exp(-|t|/t_p)$

$$E_3 \sim \nu^{-2}(1 - \nu^{-1} \arctan \nu). \tag{16}$$

This dependence is analogous to the former $(1 + \tau_e^2 \Omega^2)^{-1}$. Actually, $\nu^{-2}(1 - \nu^{-1} \arctan \nu) \sim \frac{1}{3} =$ const for $\nu \ll 1$ and $\nu^{-2}(1 - \nu^{-1} \arctan \nu) \approx \nu^{-2}$ for $\nu \gg 1$. The dependence $\log[\nu^{-2}(1 - \nu^{-1} \arctan \nu)]^{1/2}$ is shown in fig. 3.

It is worth to note that it is possible also to record the signal intensity J_3 (Ω, t) by time-resolved technics.

Thus, the light-induced moving grating method permits direct observation of the dependence of the rate of thermal electron relaxation on temperature τ_e^{-1} (T_0) unlike pump-probe methods [3,4,6–9].

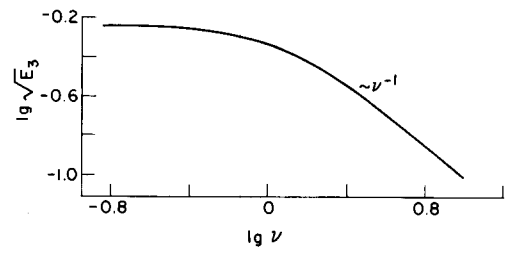

Fig. 3. The dependence of signal energy $E_3 \sim \nu^{-2}(1 - \nu^{-1} \arctan \nu)$ for two-sided exponential exciting pulses.

3. Conclusion

In this work we proposed a new method for the investigation of EP interaction in metals and superconductors with femtosecond time resolution. We have developed the theory of the method for both weak and strong heating electron system regimes. The method is based on laser-induced moving gratings and unlike the pump-probe methods [3,4,6–9] does not require complex and expensive femtosecond laser systems. In addition, in the weak heating regime our method does not require the knowledge of the lattice and electron specific heats C_i and C_e, respectively, and also the initial and the final electron temperatures $T_e(0)$ and $T_e(\infty)$. The expression (13) for the determination of τ_e (and, therefore, the electron–photon coupling constant (see eq. (1)) is very simple. Thus, it can be expected that our suggested method will be more accurate than the pump-probe methods. Besides, there is no impulsive excitation of nonequilibrium phonons in our method (see Introduction).

In the strong heating regime our moving grating method unlike pump-probe ones permits the direct measurement of the dependence of thermal electron relaxation on temperature.

In addition, the moving grating and the nonstationary spectroscopy methods complement each other. A complete information on a certain material can be obtained by using both methods.

The study of EP interaction in HT superconduct-

ing films is also interesting from a practical point of view since such a study will make possible to predict the properties of various detecting and switching devices [24].

Our results show that it is possible, to obtain a measurable signal with sample heating of only ~ 1 K per single laser pulse. Thus, our method allows to investigate HT superconducting samples without superconductivity destruction. In this case it will also be possible to obtain information about the dynamics of the processes of the Cooper pair destruction and formation. Corresponding theoretical study is in progress.

Acknowledgements

I thank Professor D. Huppert for useful discussions.

Appendix

A steady-state solution for eq. (14) will be sought in the form

$$T_e = \sum_{n=-\infty}^{\infty} T_n \exp[-in(\Omega t - \boldsymbol{q}\cdot\boldsymbol{r})] .$$

Substituting the assumed solution into eq. (14), we obtain algebraic equations for harmonics T_n

$$T_0 = F_0/\alpha , \qquad T_1 = T_0^{-1}[\tau_e^{-1}(T_0) - i\Omega]^{-1} \times \left(F_1/\bar{\gamma} + i\Omega \sum_{k=1}^{\infty} T_k^* T_{k+1} \right), \tag{17}$$

$$T_p = i\Omega T_0^{-1}[p^{-1}\tau_e^{-1}(T_0) - i\Omega]^{-1} \times \left(\sum_{k=1}^{\infty} T_k^* T_{k+p} + \tfrac{1}{2} T_{p/2}^2 + \sum_{k=1}^{(p/2)-1} T_k T_{p-k} \right), \quad p = 2m , \tag{18}$$

$$T_p = i\Omega T_0^{-1}[p^{-1}\tau_e^{-1}(T_0) - i\Omega]^{-1} \times \left(\sum_{k=1}^{\infty} T_k^* T_{k+p} + \sum_{k=1}^{(p-1)/2} T_k T_{p-k} \right), \quad p = 2m+1 , \quad m = 1, 2, \ldots . \tag{19}$$

As it can be seen from eqs. (17)–(19), the harmonics T_p with $p \geqslant 2$ will be small for both large ($\Omega \gg \tau_e^{-1}(T_0)$) and small ($\Omega \ll \tau_e^{-1}(T_0)$) frequency detunings Ω. Therefore, it is possible to disregard them for these cases.

Disregarding T_p with $p \geqslant 2$, we obtain the partial solution (15) for eq. (14) from eqs. (17)–(19). The stability of periodic solution (15) depends on that of a linearized equation which is satisfied by small variations δT_e [25]. So, we shall suppose $T_e'(t) = T_e(t) + \delta T_e(t)$ and shall substitute for T_e by this expression in eq. (14). We obtain

$$\frac{d\delta T_e}{dt} = \frac{1}{T_e}\left(\frac{\alpha}{\gamma} + \frac{\partial T_e}{\partial t} \right) \delta T_e \equiv Q\, \delta T_e . \tag{20}$$

Eq. (20) is the linear differential equation with periodic coefficients having the same period T'' as the given solution $T_e(t)$. The solution for eq. (20) can be written in the form

$$\delta y = \text{const} \times \exp\left(\int_0^{T''} Q\, dt \right).$$

The periodic solution is stable if $(1/T'')\int_0^{T''} Q\, dt < 0$ [25]. Calculating the integral $\int_0^{T''} Q\, dt$ by the methods of the theory of the functions of complex variable, we obtain

$$\frac{1}{T''}\int_0^{T''} Q\, dt = -\tau_e^{-1}(T_0)/a < 0 , \qquad 0 < a < 1 ,$$

$$a = [1 - 4|F_1/F_0|^2 \tau_e^{-2}(\tau_e^{-2} + \Omega^2)^{-1}]^{1/2} .$$

Thus, the partial solution (15) for eq. (14) is stable.

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